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Path-Integral and BRST Quantization in a Pure Supersymmetric Anyon Model

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The constraint structure of a pure supersymmetric anyon model with $U(1)$ gauge symmetry is analyzed in the framework of the symplectic Faddeev–Jackiw formalism. Then the path-integral method is used to develop the perturbative formalism. The Feynman rules and the diagrammatics are given. Finally, the results are compared to those obtained by means of the BRST formalism.

1. INTRODUCTION

Anyonic excitations can be analyzed from different theoretical approaches (see, for instance, refs. 1–14), but a natural way to treat fractional spin and statistics is by means of supersymmetric anyon models.

In the framework of gauge theories, an interesting supersymmetric formulation of pure anyon theories is presented in ref. 13. Starting from the standard formalism of pure anyon theories in terms of the $U(1)$ statistical Chern–Simons (CS) field, a minimal supersymmetric model with fractional spin and statistics is constructed. This is done by direct generalization of the nonsupersymmetric case. The first important result is that the fields of spin s are connected with the fields of spin $s + \frac{1}{2}$ by means of the supersymmetry. When the particle content and the interactions of the model are explored, an anyon–anyon interaction required by supersymmetry naturally appears. So the main conclusion is that in this type of model the interaction among anyons is a direct requirement of supersymmetry. This fact can be seen from a general point of view by defining a minimal coupling among a suitably conserved current superfield and a gauge spinor superfield. There are several reasons and advantages in considering anyon theories in the supersymmetry frame-

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work. For instance, when the problem of spontaneous symmetry breaking is treated, both the supersymmetry and the internal gauge symmetries are relevant, and so the different physical meaning of both symmetry breakings are always present.

The purpose of this paper is to study a supersymmetric anyon model in the framework of the perturbative quantum theory by giving the Feynman rules and diagrammatics of the supersymmetric model. The first step is to study the constraint structure. This can be done by using the usual Dirac method for Hamiltonian constrained systems.[15]

An alternative way to treat constrained systems is by using the symplectic Faddeev–Jackiw (FJ) Lagrangian method.[16] This seems to be more economical when it is applied to a supersymmetric model having several constraints because it involves a minor number of constraints.

The FJ symplectic formalism has been studied carefully by several authors.[17–23] The supersymmetric generalization of the FJ symplectic formalism including Grassmann field dynamical variables was given in refs. 24 and 25, but is not often used in supersymmetric systems.

Recently[26] the key equations of the supersymmetric extension of the FJ symplectic formalism were written in such a way that the inverse of the symplectic matrix is easily computed. The results were applied to study the constraint structure of supersymmetric anyon models.

In this paper, by using our results, we carry out the quantization of a pure supersymmetric anyon system by means of the path-integral formalism.

The paper is organized as follows: In Section 2, we briefly describe the classical supersymmetric Lagrangian theory for pure anyon systems by writing the constraints provided by the FJ formalism. In Section 3, we construct the perturbative formalism by using the path-integral method.[27] The Feynman rules and the diagrammatics are found. Finally, in Section 4, the supersymmetric model is analyzed from the Becchi–Rouet–Stora–Tyupin (BRST) formalism.[28]

2. PRELIMINARIES AND CLASSICAL SUPERSYMMETRIC LAGRANGIAN

In this section, the main results of ref. 26 are used in order to determine the supersymmetric Lagrangian density and the constraint structure.

As shown in Section 3 of ref. 26, the starting point is to consider the minimal supersymmetric action for pure anyon theories by using scalar superfields in the superspace of coordinates (x_μ, θ_α) $(\theta = 0, 1, 2)$, where θ_α $(\alpha = 1, 2)$ is a Majorana spinor. Once the action is written, the integration on the Grassmann variables θ_α can be performed and the minimal supersymmetric

Lagrangian density in components, in the Wess–Zumino (WZ) gauge, reads as follows:

$$\mathcal{L}_{WZ} = D_\mu \varphi^* D^\mu \varphi - m^2 \varphi^* \varphi + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi + ie\,(\bar{\psi}\lambda\varphi - \bar{\lambda}\psi\varphi^*)$$
$$+ \frac{e^2}{4\pi\vartheta}\,\varepsilon_{\mu\nu\rho}\,A^\mu\,\partial^\nu\,A^\rho - \frac{e^2}{4\pi\vartheta}\,\bar{\lambda}\lambda \tag{2.1}$$

where A_μ is the statistical field with gauge symmetry $U(1)$, ϑ is the statistical parameter, and $D_\mu = \partial_\mu - ieA_\mu$ is the covariant derivative. In equation (2.1), φ is a complex scalar field, ψ is a Dirac spinorial field, and λ_α (gaugino) is a Majorana spinor, superpartner of A_μ.

The convention used is $\varepsilon^{012} = \varepsilon^{12} = 1$, and the Minkowskian metric is $g_{\mu\nu} = \mathrm{diag}(1, -1, -1)$. In this paper, we use the $(2 + 1)$-dimensional representation of the Clifford algebra with the Dirac γ-matrices $\gamma^0 = \sigma^2$, $\gamma^1 = i\sigma^1$, and $\gamma^2 = i\sigma^3$, where the σ^i are the Pauli matrices.

In order to simplify algebraic manipulations in the Lagrangian density (2.1), we do not write the gauge kinetic term $(-\frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A))$ for the statistical field A_μ and the corresponding kinetic term $(\frac{1}{2}i\lambda\gamma^\mu\partial_\mu\lambda)$ for the gaugino field λ. In the absence of the kinetic term, the $U(1)$ gauge field A_μ is nondynamical as is its superpartner λ. This does not change the results because the fractional statistics described by the Lagrangian density (2.1) only depends on the presence of the CS term (and the supersymmetric partner) breaking both parity and time-reversal invariance.

Another simplification is to consider the WZ gauge. We have called the Lagrangian density (2.1) supersymmetric. Nevertheless, it is known that in the WZ gauge this property is lost.

In order to guarantee invariance under both supersymmetry and gauge tranformations, other fields which are purely geometric objects must be included. This must be taken into account in problems of symmetry breaking, but it is not necessary for our purpose.

Consequently, the Lagrangian density (2.1) describes the interacting theory of a field φ of spin $s = \vartheta/2$ and a field ψ of spin $s = \frac{1}{2} + \vartheta/2$ (and their conjugates) in a particular gauge. Moreover, when the gauge coupling is taken to zero, $e \to 0$, both the anyon behavior and the interaction terms go away, while supersymmetry remains. The supersymmetry also can be eliminated while the fractional spin and statistics are maintained. In this last case, the model is reduced to a bosonic or a fermionic anyon system. Also, it can be seen how the supersymmetry naturally leads to anyon–anyon interactions given by the fourth term of equation (2.1) through the couplings to the gaugino $\underline{\lambda}_\alpha$, superpartner of the $U(1)$ gauge field A_μ. The gaugino mass term $(e^2/4\pi\vartheta)\bar{\lambda}\lambda$ is also present, serving as gauge-invariant mass term for the gauge field A_μ.

As noted in the introduction, the information about the constraint structure can be obtained by following the usual Dirac algorithm. For the reason given above, it is convenient to use the supersymmetric extension of the symplectic FJ method given in ref. 26.

As is well known, the symplectic FJ method is formulated for actions only containing first-order time derivatives. So the starting point is to write the Lagrangian density (2.1) in first-order form as follows[26]:

$$\mathcal{L}^{(0)} = \dot{\varphi} K_\varphi + \dot{\varphi}^* K_{\varphi^*} + \dot{A}_i K_A^i + \dot{\psi}_\alpha K_{\psi_\alpha} - \mathbf{V}^{(0)} \tag{2.2}$$

where $i = 1, 2$ is a spatial index.

The symplectic potential $\mathbf{V}^{(0)}$ is given by

$$\begin{aligned}\mathbf{V}^{(0)} = & -\partial_i\varphi^*\partial^i\varphi + P_\varphi P_\varphi^* + m^2\varphi^*\varphi + m\bar{\psi}\psi - i\bar{\psi}\gamma^i\partial_i\psi - e^2\bar{A}^2(\varphi^*\varphi) \\ & + ieA_0(P_\varphi^*\varphi - P_\varphi\varphi^*) + ieA^i(\partial_i\varphi^*\varphi - \varphi^*\partial_i\varphi) \\ & - \frac{e^2}{2\pi\vartheta}\varepsilon^{ij}\partial_i A_j A_0 - ie(\bar{\psi}\lambda\varphi - \bar{\lambda}\psi\varphi^*) + \frac{e^2}{4\pi\vartheta}\bar{\lambda}\lambda\end{aligned} \tag{2.3}$$

and the coefficients in the symplectic part of (2.2) are

$$K_\varphi = P_\varphi^* = \dot{\varphi}^* + ieA_0\varphi^* \tag{2.4a}$$

$$K_{\varphi^*} = P_\varphi = \dot{\varphi} - ieA_0\varphi \tag{2.4b}$$

$$K_A^i = \frac{e^2}{4\pi\vartheta}\varepsilon^{ij}A_j \tag{2.4c}$$

$$K_{\psi_\alpha} = \psi_\alpha \tag{2.4d}$$

$$K_{A_0} = 0 \tag{2.4e}$$

$$K_\lambda = 0 \tag{2.4f}$$

Therefore, the initial set of symplectic variables defining the extended configuration space of the dynamical system is given by (φ, P_φ^*, φ^*, P_φ, A_μ, ψ, λ). As shown in ref. 26, the symplectic supermatrix $M_{AB}^0\,(x, y)$ is singular. From the explicit expression of the symplectic supermatrix $M_{AB}^0\,(x, y)$ it can be seen that according to the singular symplectic variables A_0 and λ the following two constraints can be found:

$$\Omega_1 = ie(P_\varphi^*\varphi - P_\varphi\varphi^*) - \frac{e^2}{2\pi\vartheta}\varepsilon^{ij}\partial_i A_j = 0 \tag{2.5a}$$

$$\Omega_{2\alpha} = e(\varphi - \varphi^*)(\gamma_0\psi)_\alpha + \frac{ie^2}{2\pi\vartheta}(\gamma_0\lambda)_\alpha = 0 \tag{2.5b}$$

Equation (2.5a) is the A_0 equation of motion (bosonic constraint), and equation (2.5b) is the λ equation of motion (fermionic constraint). These two constraints form a supermultiplet. These are algebraic equations on the component A_0 field and the nonpropagating λ field, respectively, and can be used to eliminate them.

Subsequently, by carrying out the first iterative procedure, the expression for the first-iterated Lagrangian reads

$$\mathcal{L}^{(0)} \to \mathcal{L}^{(1)} = \dot{\varphi} P_{\varphi}^{*} + \dot{\varphi}^{*} P_{\varphi} + \dot{A}_i K_A^i + \dot{\psi}_{\alpha} K_{\psi_{\alpha}} + \dot{\xi}_1 \Omega_1 + \dot{\xi}_2^{\alpha} \Omega_{2\alpha} - \mathbf{V}^{(1)} \tag{2.6}$$

where $\mathbf{V}^{(1)}$ is defined by

$$\begin{aligned} \mathbf{V}^{(1)} &= \mathbf{V}^{(0)} \Big|_{\Omega_1 = \Omega_2 = 0} \\ &= -\partial_i \varphi^* \partial^i \varphi + P_{\varphi} P_{\varphi}^{*} + m^2 \varphi^* \varphi + m \bar{\psi} \psi - i \bar{\psi} \gamma^i \partial_i \psi - e^2 \bar{A}^2 (\varphi^* \varphi) \\ &\quad + ie A^i (\partial_i \varphi^* \varphi - \varphi^* \partial_i \varphi) + \frac{\pi}{e^2} \vartheta (\varphi - \varphi^*)^2 \bar{\psi} \psi \end{aligned} \tag{2.7}$$

The extended configuration space is now defined by the set of variables $(\varphi, P_{\varphi}^{*}, \varphi^*, P_{\varphi}, A_i, \psi, \lambda, \xi_1, \xi_2)$.

By continuing the FJ algorithm, it is easy to prove that no new constraints appear.

On the other hand, it is known that the FJ algorithm is unable to produce an invertible symplectic matrix when it is applied to gauge field theories in which true first-class constraints exist.

The invertibility is obtained by means of a gauge-fixing term we must add to the classical Lagrangian density in order to break the gauge symmetry of the symplectic potential. The simplest case is to consider the Coulomb gauge for the $U(1)$ statistical gauge field A_{μ},

$$F = \partial_i A^i = 0 \tag{2.8}$$

Once the gauge-fixing condition (2.8) is given, the symplectic supermatrix is invertible, thus providing the generalized FJ graded brackets in such a particular gauge, which correspond to the graded Dirac brackets of the model. Therefore, from the canonical point of view the dynamical information provided by the symplectic FJ formalism is equivalent to that obtained by means of the Dirac formalism for constrained systems.

Finally, from a simple algebraic computation it is easy to show that the following linear combination of constraints is the only true constraint in this model:

$$\Omega = e\left(\partial_i P_A^i - \frac{e^2}{4\pi\vartheta}\epsilon^{ij}\,\partial_i A_j\right) = 0 \tag{2.9}$$

Consequently, in a path-integral quantization formalism the construction of the diagrammatics can be done by only considering this last constraint, called in the Dirac language a first-class constraint.

3. PATH-INTEGRAL QUANTIZATION AND PERTURBATIVE METHOD

In this section, we implement the perturbative method by constructing the diagrammatics in the framework of the path-integral method according to the Faddeev formalism for systems with first-class constraints.[29] The partition function for the $U(1)$ gauge supersymmetric model contains the path integration on all the variables of the extended configuration space. Performing the Gaussian integrals over the P_φ and P_φ^* variables, we assume that the starting partition function can be written as follows:

$$Z = \int \prod \mathfrak{D}(A_i)\mathfrak{D}(\varphi^*)\mathfrak{D}(\varphi)\mathfrak{D}(\bar{\psi})\mathfrak{D}(\psi)\delta(F)$$
$$\times \det[\Omega, F]\exp\left(i\int d^3x\,\mathcal{L}_{\text{eff}}\right) \tag{3.1}$$

where we have called $\mathcal{L}_{\text{eff}}$ the original Lagrangian density (2.2). Moreover, in order to obtain equation (3.1), the integral representation $\delta(\Omega) = \int \mathfrak{D}\Lambda \exp\,(i \int d^3\,x\,\Lambda\Omega)$ was used.

Therefore, by taking into account the arbitrariness of the multiplier Λ and following the usual steps, it is possible to rescale the corresponding integration variable in such a way as to recover the original Hamiltonian density or symplectic potential.

In equation (3.1), the function $\det[\Omega, F]$ is written as follows:

$$\det[\Omega, F] = e\nabla^2\delta(x - y) \neq 0 \tag{3.2}$$

As this determinant does not depend on the field variables, it is included in the path-integral normalization factor.

Finally, we use the Faddeev–Popov trick to go over to a general covariant gauge by writing the gauge-fixing condition (2.8) in the form $\partial_\mu A^\mu$ −

$c(x) = 0$, where $c(x)$ is an arbitrary function. So we find the final expression of the partition function (3.1):

$$Z = \int \mathcal{D}A_i \mathcal{D}\varphi \mathcal{D}\varphi^* \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(i \int d^3x \mathcal{L}^* \right) \tag{3.3}$$

In this equation, the functional $\mathcal{L}^*$ is given by

$$\mathcal{L}^* = \mathcal{L}_{\text{eff}} + \mathcal{L}_{\text{fix}} \tag{3.4}$$

where

$$\mathcal{L}_{\text{fix}} = \frac{\Gamma}{2} (\partial_\mu A^\mu)^2 \tag{3.5}$$

where Γ is a Lagrange multiplier.

Looking at equation (3.3), we can see that a fruitful form for the partition function is obtained. The quantum problem is described in terms of a path integral over all the independent field components of the model: A_i, φ, φ^*, $\bar{\psi}$, and ψ. Subsequently, the problem can be treated using the diagrammatics technique in the framework of the Feynman path-integral perturbation theory. In principle, it is straightforward to go from the path integral (3.3) to the Feynman rules for propagators and vertices.[27]

So we recognize the quadratic parts of the Lagrangian density $\mathcal{L}^*$ as representing the propagators and the remaining pieces as representing the vertices. Consequently, $\mathcal{L}^*$ defines the effective Lagrangian density of the anyonic system under consideration and it can be partitioned as follows:

$$\begin{aligned} \mathcal{L}^* = {} & \mathcal{L}^*(A_\mu) + \mathcal{L}^*(\varphi^*, \varphi) + \mathcal{L}^*(\bar{\psi}, \psi) \\ & + \mathcal{L}^*_{\text{int}} (A_\mu, \varphi^*, \varphi, \bar{\psi}, \psi) \end{aligned} \tag{3.6}$$

We have denoted

$$\mathcal{L}^*(A_\mu) = \tfrac{1}{2} A_\mu (D^{-1})^{\mu\nu} A_\nu \tag{3.7a}$$

$$\mathcal{L}^*(\varphi^*, \varphi) = \varphi^* P^{-1} \varphi \tag{3.7b}$$

$$\mathcal{L}^*(\bar{\psi}, \psi) = \bar{\psi} G^{-1} \psi \tag{3.7c}$$

$$\begin{aligned} \mathcal{L}^*_{\text{int}}(A_\mu, \varphi^*, \varphi, \bar{\psi}, \psi) = {} & e^2 \varphi^* A_\mu V^{\mu\nu} A_\nu \varphi \\ & - 2\pi\vartheta[\bar{\psi}\psi (\varphi - \varphi^*)\varphi - \bar{\psi}(\varphi - \varphi^*)\psi\varphi^*] \\ & + 2ie\varphi^* A_\mu \partial^\mu \varphi + e\bar{\psi}\gamma_\mu A^\mu \psi \end{aligned} \tag{3.7d}$$

In equation (3.7a), the 3×3 matrix D^{-1} is the inverse of the propagator associated to the field A_μ. So the propagator $D_{\mu\nu}(k)$ in the momentum space can be evaluated as

$$D_{\mu\nu}(k) = \frac{(-i)2\pi\vartheta}{e^2\Gamma k^4}\left[\left(\frac{ie^2}{2\pi\vartheta}\right)k_\mu k_\nu + \Gamma k^2 k^\rho\,\varepsilon_{\mu\nu\rho}\right] = \frac{k_\mu k_\nu}{\Gamma\,k^4} - \frac{2\pi i\vartheta}{e^2k^2}\,k^\rho\varepsilon_{\mu\nu\rho} \tag{3.8}$$

Furthermore, in (3.7b) and (3.7c), P^{-1} and G^{-1} are the inverses of the propagators associated to the bosonic and fermionic anyonic matter fields, respectively. In the momentum space, these propagators are given by

$$P(l) = \frac{1}{l^2 - m^2} \tag{3.9}$$

$$G(p) = \frac{i(\gamma\cdot p + m)}{p^2 - m^2} \tag{3.10}$$

respectively.

Finally, equation (3.7d) is the part of the Lagrangian density which accounts for the vertices of the model. There are two four-leg vertices. In this equation, the 3×3 matrix $V^{\mu\nu}$ is written

$$V^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.11}$$

The other vertices have three legs, one of which is derivative.

Now we can write the Feynman rules for propagators and vertices.

(i) *Propagators* : We associate with the propagator $D_{\mu\nu}(k)$ of the bosonic field A_μ a wavy line connecting two generic points

We associate with the usual propagators of the bosonic $P(l)$ and the fermionic $G(p)$ matter fields a dashed and a continuous line

- - - - - - - - - - - - - -

$l \rightarrow$

and

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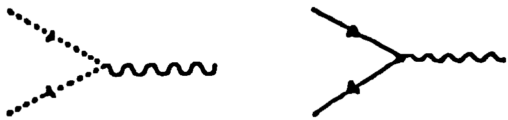
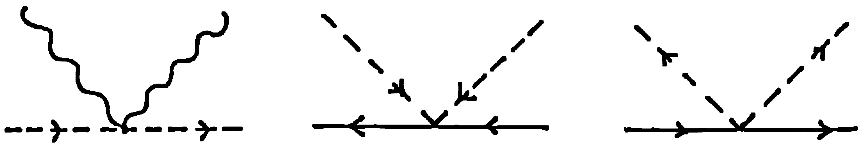
$p \rightarrow$

respectively.

(ii) *Vertices*: The three-leg vertices $2ie\partial^{\mu}$ and $e\gamma_{\mu}$ of the model are standed for



respectively. The four-leg vertices $e^2 V^{\mu\nu}$, $2\pi\vartheta$, and $-2\pi\vartheta$ are standed for



respectively.

Moreover, as usual, we have to take into account a minus sign for every closed fermion loop and another minus sign for diagrams related to the exchange of two fermion lines, internal or external. A combinatorial factor correcting for double counting in the case that identical particles occur also must be considered.

We do not treat here the problem of regularization and renormalization of this model. However, by looking at the expressions of the propagators and vertices and taking into account the above Feynman rules, complete information about the perturbative behavior could be obtained. At least the one-loop structure can be easily studied by analyzing the superficial degree of divergence of the corresponding diagrams. It can be seen that this gauge model belongs to the class of theories with only a finite number of divergent diagrams. So the regularization and renormalization problem is reduced to the problem of regularizing a superrenormalizable theory and it can be done by the usual methods.

4. THE BRST FORMALISM

We are going to construct the BRST formalism for the constrained Hamiltonian system under consideration by using most of the tools of ref. 28.

From the canonical point of view the quantized Hamiltonian system is defined by the first-class constraint Ω given in equation (2.9), the Dirac graded brackets for the dynamical variables, and the Hamiltonian (or symplectic potential in the FJ language). Therefore, all the graded brackets we write hereafter are understood as Dirac graded brackets (or generalized FJ brackets).

Furthermore, since $\mathbf{V}^{(1)}$ and Ω are first-class quantities, the more general first-class Hamiltonian density for this model can be written as

$$\mathbf{V}^{(1)*} = \mathbf{V}^{(1)} + u\Omega \tag{4.1}$$

where u is a Lagrange multiplier.

Therefore, the following equations hold:

$$[\Omega, \Omega]_- = C\Omega = 0 \tag{4.2a}$$

$$[H, \Omega]_- = V\Omega = 0 \tag{4.2b}$$

where $H = \int d^2x\mathbf{V}^{(1)}$.

From (4.2a), trivially we note that C vanishes. Furthermore, from (4.2b), we note that in the constrained Hamiltonian system under consideration, also the coefficient V vanishes for a suitable choice of the Hamiltonian density $\mathbf{V}^{(1)}$, as occurs in any usual CS theory.

As is well known, in the BRST formalism it is convenient to treat the Lagrange multiplier u defined in equation (4.1) on the same footing as the remaining dynamical variables and to associate with them a canonical momentum variable $\mathcal{P}$ such that

$$[u, \mathcal{P}]_- = 1 \tag{4.3}$$

If classically the momentum is constrained to vanish, we can be sure that the dynamical structure of the theory does not change. Precisely, the first-class constraint $\mathcal{P} = 0$ generates the gauge transformation $u \to u + v$ of the multiplier, making evident their arbitrariness.

Consequently, from now on our set of variables will be

$$A_\Xi = (A_i, \varphi, \varphi^*, \bar{\psi}, \psi, u) \tag{4.4}$$

where the compound index Ξ runs over the components of the field variables.

The set of canonical conjugate momenta corresponding to the field variables is written as

$$P_\Xi = (P^i_A, P^*_\varphi, P_\varphi, \Pi_\psi, \overline{\Pi}_\psi, \mathcal{P}) \tag{4.5}$$

and the new first-class constraints are given by

$$G_A = (\Omega, \mathcal{P}), \qquad A = 1, 2 \tag{4.6}$$

Therefore, the equations (4.2) take the form

$$[G_A, G_B]_- = C^C_{AB}G_C = 0 \tag{4.7a}$$

$$[H, G_A]_- = V^B_A G_B = 0 \tag{4.7b}$$

with $C^C_{AB} = V^B_A = 0$.

Now, the BRST-invariant gauge-fixed Hamiltonian must be introduced by considering the fermionic ghost fields (Majorana spinors) η_A and their canonical conjugate momenta Π^A. So the invariant gauge-fixed Hamiltonian H_χ reads

$$H_\chi = \mathbf{V}^{(1)} + [\chi, Q]_+ \tag{4.8}$$

where $\chi = \Pi_B \omega^B$, with ω^B being the gauge-fixing conditions given by the set of quantities

$$\omega^B = -(u, F) \tag{4.9}$$

Furthermore, we assume that in the $U(1)$ gauge supersymmetric model we are considering, the BRST generator Q is given by the well-known expression

$$Q = G_A \eta^A \tag{4.10}$$

Looking at the constraint (4.6), we can see that it can be partitioned into two subsets; therefore, we also assume that the ghosts and the antighosts are introduced in such a way that

$$\eta_A = (\eta, \Pi) \tag{4.11a}$$

$$\Pi^A = (\overline{\Pi}, \overline{\eta}) \tag{4.11b}$$

Therefore, the following canonical brackets hold:

$$[\eta, \overline{\Pi}]_+ = 1 \tag{4.12a}$$

$$[\Pi, \overline{\eta}]_+ = 1 \tag{4.12b}$$

So is easy to see that the expression for the Hamiltonian density $\mathcal{H}_\chi$ reads

$$\mathcal{H}_\chi = \mathbf{V}^{(1)} - \overline{\Pi}\Pi - \Omega u - \mathcal{P}F - \overline{\eta}[F, \Omega]_- \eta \tag{4.13}$$

When an integration in the last term of equation (4.13) is performed and since $[F(x), \Omega(y)]_- = -e\,\nabla^2\delta(x - y)$ [see equation (3.2)], the last term is written $e\overline{\eta}\nabla^2\eta$.

Consequently, the BRST Lagrangian density $\mathcal{L}^{\mathrm{BRST}}$ is given by

$$\begin{aligned} \mathcal{L}^{\mathrm{BRST}} =& \dot{A}_i P^i_A + P_\varphi \dot{\varphi}^* + P^*_\varphi \dot{\varphi} + \Pi_\psi \dot{\overline{\psi}} + \overline{\Pi}_\psi \dot{\psi} + \mathcal{P}\dot{u} \\ &+ \overline{\Pi}\dot{\eta} + \overline{\eta}\dot{\Pi} - \mathcal{H}_\chi \end{aligned} \tag{4.14}$$

When the constrained system has only first-class constraints as in the present case, the partition function in the BRST formalism is written by means of the following path integral:

$$Z = \int \mathcal{D}A_{\Xi}\, \mathcal{D}P^{\Xi}\, \mathcal{D}\eta\mathcal{D}\overline{\eta}\mathcal{D}\Pi\mathcal{D}\overline{\Pi} \exp\left(i \int d^3x \mathcal{L}^{\mathrm{BRST}} \right) \qquad (4.15)$$

This last expression for the partition function is equivalent to those obtained by means of the Faddeev procedure, which was the starting expression used above [equation (3.1)] when the diagrammatics was constructed. We conclude that the two methods give the same basic results and therefore they can be considered as alternatives.

5. CONCLUSIONS

The pure supersymmetric anyon model was treated in the framework of the FJ symplectic formalism. The constraint structure was analyzed and the first-class constraint associated with the $U(1)$ symmetry was found. A simple gauge-fixing condition compatible with this constraint was proposed.

Next, by going over to the path-integral quantization method, the partition function was written. The gauge-fixing condition allows us to determine the gauge-fixing part of the effective action. Furthermore, in the framework of the perturbative formalism, the Feynman rules and the diagrammatics of the pure supersymmetric anyon model are constructed. By means of the propagators thus defined, all the diagrams are obtained by connecting vertices and sources as usual.

The coupled system has three-leg and four-leg vertices. The vertex structure is a direct consequence of the coupling properties of the supersymmetric Lagrangian.

Moreover, looking at the diagrammatics, it is possible to conclude that the model belongs to the class of superrenormalizable theories because it has a finite number of divergent diagrams.

As briefly noted but not shown, by using the perturbative formalism developed here all the information and prescriptions about the regularization and renormalization of the model can be given.

In the last section, by using well-known tools and methods, the BRST formalism of the gauge supersymmetric model was given. It was shown that the partition function obtained from the BRST formalism is equivalent to that obtained by means of the Faddeev method, as expected.

Finally, the results obtained by application of the FJ symplectic method and those obtained from the usual Dirac formalism are equivalent. Nevertheless, the algebraic manipulations in the FJ symplectic method are shorter than in the Dirac procedure.

REFERENCES

1. F. A. Berezin and M. S. Marinov, *Ann. Phys.* (NY) **104**, 336 (1977); J. M. Leinass and J. Myrheim, *Nuovo Cimento* **37**, 1 (1977).
2. G. Goldin, R. Menikoff, and D. Sharp, *J. Math. Phys.* **21**, 650 (1980); **22**, 1664 (1981).
3. F. Wilczek, *Phys. Rev. Lett.* **49**, 957 (1982).
4. F. Wilczek and A. Zee, *Phys. Rev. Lett.* **51**, 2250 (1983).
5. F. Wilczek, *Fractional Statistics and Anyon Superconductivity*, World Scientific, Singapore (1991).
6. R. B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983); see also articles in S. S. Chern, C. W. Chu, and C. S. Ting, eds., *Physics and Mathematics of Anyons*, World Scientific, Singapore (1991).
7. Y. S. Wu and A. Zee, *Phys. Lett. B* **147**, 325 (1984).
8. M. J. Bowick, D. Karabali, and L. C. R. Wijewardhana, *Nucl. Phys. B* **271**, 417 (1986).
9. I. Dzyaloshinskii, A. Polyakov, and P. Wiegmann, *Phys. Lett. A* **127**, 112 (1988).
10. A. M. Polyakov, *Mod. Phys. Lett. A* **3**, 325 (1988).
11. M. S. Plyushchay, *Int. J. Mod. Phys. A* **7**, 7045 (1992).
12. C. R. Hagen, *Ann. Phys.* (NY) **157**, 342 (1984); *Phys. Rev. D* **31**, 848 (1985); **31**, 2135 (1985); D. Arovas, J. Schrieffer, F. Wilczek, and A. Zee, *Nucl. Phys. B* **251**, 117 (1985); for review see R. Jackiw, in *Physics, Geometry and Topology*, H. C. Lee, ed., Plenum, New York (1990).
13. Z. Hlousek and D. Spector, *Nucl. Phys. B* **344**, 793 (1990).
14. A. Foussats, E. Manavella, C. Repetto, O. P. Zandron, and O. S. Zandron, *Int. J. Mod. Phys. A* **11**, 921 (1996).
15. P. A. M. Dirac, *Can. J. Math.* **2**, 129 (1950); *Lectures on Quantum Mechanics*, Yeshiva University Press, New York (1964).
16. L. Faddeev and R. Jackiw, *Phys. Rev. Lett.* **60**, 1692 (1988).
17. M. V. E. Costa and H. O. Girotti, *Phys. Rev. Lett.* **60**, 1771 (1988).
18. J. Barcelos-Neto and P. P. Srivastava, *Phys: Lett. B* **259**, 456 (1991).
19. D. S. Kulshreshtha and H. J. W. Muller-Kirsten, *Phys. Rev. D* **43**, 3376 (1991).
20. J. Barcelos-Neto and C. Wotzasek, *Int. J. Mod. Phys. A* **7**, 4981 (1992).
21. J. Barcelos-Neto and C. Wotzasek, *Mod. Phys. Lett. A* **7**, 1172 (1992).
22. M. M. Horta-Barreira and C. Wotzasek, *Phys. Rev. D* **45**, 1410 (1992).
23. H. Montani and C. Wotzasek, *Mod. Phys. Lett. A* **8**, 3387 (1993).
24. J. Govaerts, *Int. J. Mod. Phys. A* **5**, 3625 (1990).
25. A. Foussats and O. S. Zandron, *J. Phys. A* **30**, L513 (1997).
26. A. Foussats, C. Repetto, O. P. Zandron, and O. S. Zandron, *Ann. Phys.* (NY) **268**, 225 (1998).
27. G. 't Hooft and M. Velman *Diagramar*, CERN, Geneva (1973).
28. C. Becchi, A. Rouet, and R. Stora, *Ann. Phys.* (NY) **98**, 287 (1976); E. S. Fradkin and G. A. Vilkovisky, *Phys. Lett. B* **55**, 224 (1975); M. Henneaux, *Phys. Rep.* **126**, 1 (1985); R. Marnelius, Introduction to the quantization of general gauge theories, Institute of Theoretical Physics S-412 96, Göteborg Sweden (1981); E. S. Fradkin and T. E. Fradkina, *Phys. Lett. B* **72**, 343 (1978); I. V. Tyupin, Lebedev Preprint FIAN 39, unpublished [in Russian].
29. L. D. Faddeev, *Theor. Math. Phys.* **1**, 1 (1970).